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ON LIBRATION POINTS NEAR A GRAVITATING AND ROTATING TRIAXIAL ELLIPSOID*

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The problem of the motion of a passively gravitating point near the relative equilibrium position around a uniformly rotating ellipsoid with principal semiaxes of arbitrary lengths is examined. A system of parameters defining the problem, in which the domain wherein the necessary stability conditions are fulfilled, is selected. Certain qualitative properties of the mechanical system being examined are revealed by an analysis of the domain's geometric structure.

The problem of a satellite's motion in a neighborhood of the relative equilibrium position near a rotating planet having the form of a triaxial ellipsoid was examined in /1/ in a linear setting and in /2/ in a nonlinear one. The ellipsoid was assumed to be nearly a ball, which corresponds to real planets. However, objects having a clearly expressed ellipsoidal form do exist (for example, elliptic galaxies); therefore, the problem of the motion of a satellite in a neighborhood of the libration points of a rotating ellipsoid with principal semiaxes of arbitrary lengths is of definite interest. In addition, an investigation of the problem in a general setting permits a sufficiently rapid obtaining of qualitative conclusions on the mechanical properties of the motion for a planet close in form to a ball.

1. Statement of the problem. Assume that a homogeneous ellipsoid of mass M with principal axes OA, OB, OC of lengths a, b, c respectively (Fig.1), rotates uniformly around the axis OC with angular velocity ω. We choose a coordinate system with axes Ox, Oy, Oz directed towards A, B, C respectively.

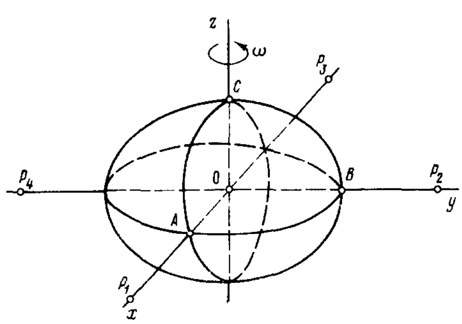

Fig.1

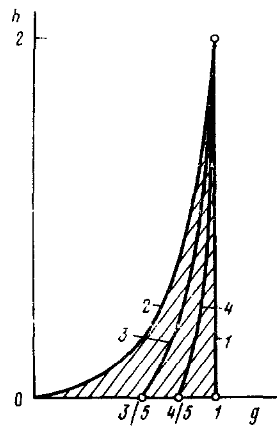

Fig.2

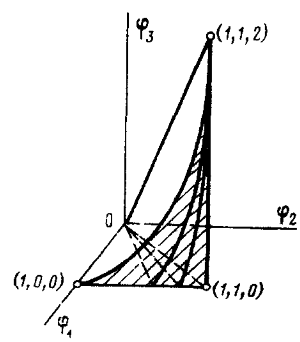

Fig.3

We introduce the charateristic length l by the relations $a = l\alpha$, $b = l\beta$, $c = l\gamma$, $\alpha^2 + \beta^2 + \gamma^2 = 1$. The triple (α, β, γ) specifies the representative point on a spherical triangle S_+^2 defined by the inequalities $\alpha > 0$, $\beta > 0$, $\gamma > 0$. As the chracteristic time we take the ellipsoid's rotation period around its axis and we introduce dimensional variables by the formulas

$$t = \tau / \omega,\ x = lq_1,\ y = lq_2,\ z = lq_3$$

The problem's parameter space can be defined as the set of pairs

$$\Pi^3 = \{(s,\ \rho)\},\ s \in S_+^2,\ \rho \in \mathbf{R}_+^1 = \{\rho : \rho > 0\},\ \rho = 3fM / (4\omega^2 l^3)$$

It is natural to identify it with the $\mathbf{R}_+^3$-octant in $\mathbf{R}^3$

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$$\mathbf{R}_+^3 = \{(s_1, s_2, s_3) : s_1 = \rho\alpha,\ s_2 = \rho\beta,\ s_3 = \rho\gamma\}$$

Using the Legendre transformation we arrive at the canonic phase variables (p_1, p_2, p_3). As a result we obtain the system of Hamiltonian equations

$$q_i^\cdot = \partial H / \partial p_i,\ p_i^\cdot = -\partial H / \partial q_i,\ i = 1, 2, 3$$

$$H(\mathbf{q}, \mathbf{p}) = \frac{1}{2}(p_1^2 + p_2^2 + p_3^2) + (p_1 q_2 - p_2 q_1) - \rho \int_\mu^\infty \left(1 - \frac{q_1^2}{\alpha^2 + u} - \frac{q_2^2}{\beta^2 + u} - \frac{q_3^2}{\gamma^2 + u}\right) \frac{du}{S(u)}$$

$$S(u) = [(\alpha^2 + u)\ (\beta^2 + u)\ (\gamma^2 + u)]^{1/2},\quad q_1^2 / (\alpha^2 + \mu) + q_2^2 / (\beta^2 + \mu) + q_3^2 / (\gamma^2 + \mu) = 1$$

2. Libration points. From the relations

$$\frac{\partial H}{\partial p_3} = p_3 = 0,\quad \frac{\partial H}{\partial q_3} = 2\rho q_3 F(\mu, \gamma^2),\quad F(\mu, \gamma^2) = \int_\mu^{+\infty} \frac{du}{(\gamma^2 + u) S(u)}$$

it follows that the libration points lie in the equatorial plane $q_3 = 0$, since $F(\mu, \gamma^2) > 0$ always in Π^3 and the closeness in form to a sphere, required in /5/, is not necessary. We now consider the remaining equilibrium equations under the condition $q_3 = p_3 = 0$:

$$\partial H / \partial p_1 = p_1 + q_2 = 0,\quad \partial H / \partial p_2 = p_2 - q_1 = 0$$

$$\partial H / \partial q_1 = -p_2 + 2\rho q_1 F(\mu, \alpha^2) = 0,\quad \partial H / \partial q_2 = p_1 + 2\rho q_2 F(\mu, \beta^2) = 0$$

$$q_1^2 / (\alpha^2 + \mu) + q_2^2 / (\beta^2 + \mu) = 1$$

Two combinations of solutions are possible when $\alpha \neq \beta$ (the case $q_1 = q_2 = 0$ corresponds to a point at the ellipsoid's center and, therefore, is omitted). The coordinates of the libration points satisfy the equations

$$q_1 [1 - 2\rho F(\mu, \alpha^2)] = 0,\quad q_2 [1 - 2\rho F(\mu, \beta^2)] = 0$$

Hence

$$q_1 = 0,\ 1 - 2\rho F(q_2^2 - \beta^2, \beta^2) = 0;\ q_1 = 0,\ q_2 = \pm q_2^\circ,\quad q_2 = 0,\ 1 - 2\rho F(q_1^2 - \alpha^2, \alpha^2) = 0;\ q_1 = \pm q_1^\circ,\ q_2 = 0$$

When $\alpha = \beta$ the planet becomes an ellipsoid of revolution and the equilibrium positions are not isolated and are located on the circle $q_1^2 + q_2^2 = \alpha^2$, where μ satisfies the equation

$$1 - 2\rho \int_\mu^{+\infty} \frac{du}{(\alpha^2 + u)(\gamma^2 + u)^{1/2}} = 0$$

and the integral is computed in finite form in terms of elementary functions. By virtue of the problem's symmetry we can restrict the study only to the libration point $P_1 : q_1 = q_1^\circ,\ q_2 = q_2^\circ = 0,\ q_3 = q_3^\circ = 0$. The condition for its existence is the condition for the solvability of the equation

$$1 - 2\rho F(\mu, \alpha^2) = 0 \tag{2.1}$$

relative to μ for $\mu > 0$. It is expressed by the inequality $2\rho F(\mu, \alpha^2) > 1$ defining in $\Pi^3 = \mathbf{R}_+^3$ the subset of admissible parameters for which the existence of point P_1 is possible.

3. Stability domain. For a local investigation in a neighborhood of point P_1 we pass to new phase variables by the contact transformation $(\mathbf{q}, \mathbf{p}) \to (\mathbf{q}', \mathbf{p}')$ with the generating function

$$U = \sum_{j=1}^{3} (q_j - q_j^\circ)(p_j' + p_j^\circ),\quad p_1^\circ = 0,\quad p_2^\circ = q_1^\circ,\quad p_3^\circ = 0$$

whence

$$q_i' = \partial U / \partial p_i' = q_i - q_i^\circ,\ p_i = \partial U / \partial q_i = p_i' + p_i^\circ,\ i = 1, 2, 3$$

Treating (2.1) as an equation in ρ, enabling us to express it in the new parameter system $(\nu, \alpha, \beta, \gamma)$, we obtain the mapping

$$(\nu, \alpha, \beta, \gamma) \xrightarrow{d} (\rho, \alpha, \beta, \gamma),\ \rho^{-1} = 2F(\nu, \alpha^2)$$

diffeomorphically taking R_+^3 onto the domain of admissible values of the parameters. Thus, all points of R_+^3 are admissible for $(\nu, \alpha, \beta, \gamma)$. As ν grows from zero to $+\infty$ the magnitude of ρ also grows monotonically from $\rho_0 = [2F(0, \alpha^2)]^{-1}$ to $+\infty$ along the ray defined by the point (α, β, γ) on S_+^2. The system of parameters $(\nu, \alpha, \beta, \gamma)$ is more convenient, for example, than the fact that all the expressions for the coefficients of the power expansion of H in a neighborhood of point P_1 are explicit functions of sufficiently simple structure.

As a result of a significant amount of analytical computations we have obtained terms of second degreee in the phase variables $(\mathbf{q}', \mathbf{p}')$, occurring in the power series for the Hamiltonian. In sum they yield the quadratic form

$$H_2 = \frac{1}{2} p_1'^2 + \frac{1}{2} p_2'^2 + \frac{1}{2} p_3'^2 + p_1' q_2' - p_2' q_1' - \frac{\varphi_2 + \varphi_3}{2\varphi_1} q_1'^2 + \frac{\varphi_2}{2\varphi_1} q_2'^2 + \frac{\varphi_3}{2\varphi_1} q_3'^2$$

Here

$$\varphi_1 = F(\nu, \alpha^2),\ \varphi_2 = F(\nu, \beta^2),\ \varphi_3 = F(\nu, \gamma^2) \tag{3.1}$$

The characteristic equation of the first approximation system is

$$[\lambda^4 + \lambda^2(2\varphi_1 - \varphi_3) / \varphi_1 + (\varphi_1 + \varphi_2 + \varphi_3)(\varphi_1 - \varphi_2) / \varphi_1^2](\lambda^2 + \varphi_3 / \varphi_1) = 0$$

In the linear approximation the satellite performs normal oscillations with frequency $(\varphi_3 / \varphi_1)^{1/2}$ with respect to coordinate q_3' (this can be seen already from the expression for H_2). In order that the roots of the characteristic equation of planar motion be pure imaginary and distinct, it is necessary and sufficient to fulfil the conditions ensuring the possibility of linear normalization

$$\left(\frac{\varphi_3}{2\varphi_1} + \frac{\varphi_2}{\varphi_1}\right)^2 - 2\frac{\varphi_3}{\varphi_1} > 0, \quad \frac{\varphi_3}{2\varphi_1} < 1, \quad 1 + \frac{\varphi_3}{\varphi_1} > \left(\frac{\varphi_2}{\varphi_1}\right)^2 + \left(\frac{\varphi_2}{\varphi_1}\right)\left(\frac{\varphi_3}{\varphi_1}\right)$$

We have obtained the sequence of mappings

$$R_+^3 \xrightarrow{\varphi} R_+^3 \xrightarrow{\pi} R^2, \quad \varphi(\nu, \alpha, \beta, \gamma) = (\varphi_1, \varphi_2, \varphi_3) \in R_+^3, \quad \pi(\varphi_1, \varphi_2, \varphi_3) = (\varphi_2 / \varphi_1, \varphi_3 / \varphi_1) \in R^2$$

Denoting $\varphi_2 / \varphi_1 = g$, $\varphi_3 / \varphi_1 = h$, we see that H_2, and, hence, the first approximation system, depends only on the two parameters g and h. We note as well that $\varphi_1, \varphi_2, \varphi_3$ are homogeneous coordinates of a point in the parameter space (g, h), while π is a central projection mapping with center at the origin on the plane $\varphi_1 = 1$.

In the new parameter system the stability conditions are

$$(h / 2 + g)^2 - 2h > 0,\ h < 2,\ h(g - 1) < (1 - g)(1 + g)$$

Because $g > 0$ and $h > 0$ the last inequality is equivalent to the condition $g < 1$. The frequencies of the normal oscillations are

$$\omega_{1,2} = \{1 - h / 2 \pm [(h / 2 + g)^2 - 2h]^{1/2}\}^{1/2}, \quad 0 < \omega_2 < \omega_1$$

The stability domain obtained is shown in Fig.2. The same figure shows the resonance curves for resonances of first, second, third and fourth orders (curves *1–4*, respectively).

4. **Theorem on the stability property.** By S we denote the shaded domain in Fig.2. This is the section of the cone $\pi^{-1}(S)$ by the plane $\varphi_1 = 1$ in the space $(\varphi_1, \varphi_2, \varphi_3)$; the cone's vertex is at the origin. The stability domain in parameters $\varphi_1, \varphi_2, \varphi_3$ is shown in Fig. 3. The resonance sets are pieces of conic surfaces passing through the corresponding curves in the section $\varphi_1 = 1$ and the origin. We are not interested in the whole cone indicated, but only in the set $\Sigma = \pi^{-1}(S) \cap \varphi(R_+^3)$ which corresponds precisely with the stability domain $\varphi^{-1}(\Sigma)$ in the original parameter system $(\nu, \alpha, \beta, \gamma)$.

In order to ascertain the structure of set Σ we need to study in detail the properties of mapping φ: $R_+^3 \to \Sigma \subset R_+^3$. We fix α, β, γ and we determine the image of a ray in R_+^3 for $0 < \nu < +\infty$. This is the curve specified by the parametric Eqs.(3.1). Obviously, $\varphi_i \to 0$ as $\nu \to +\infty$ $(i = 1, 2, 3)$ and

$$d\varphi_3 : d\varphi_2 : d\varphi_1 = \frac{1}{\gamma^2 + \nu} : \frac{1}{\beta^2 + \nu} : \frac{1}{\alpha^2 + \nu}$$

for the tangent along the curve. Hence we see that the image of any ray is tangent to the straight line

$$\varphi_1 = \varphi_2 = \varphi_3 \tag{4.1}$$

as point $(0, 0, 0)$. Let us consider the spherical triangle S_+^2 defining the domain of admissible parameters α, β, γ (Fig.4); P is a represenative point on S_+^2. Domain $\varphi(\mathbf{R}_+^3)$ is symmetric relative to line (4.1) (it goes into itself under a rotation around line (4.1) by 120°). The curves (3.1) are tangent to (4.1) as $\nu \to +\infty$ and, as $\nu \to 0$, border on the surface specified by the parametric Eqs.(3.1) if in them substitute $\nu = 0$. These equations yield the mapping $\varphi^\circ: S_+^2 \to \mathbf{R}_+^3$. The points and the arcs corresponding to each other under the action of φ° are denoted by like digits on Fig.4.

Let open arcs L_1, L_2, L_3 and points R_1, R_2, R_3 (see Fig.4) bound S_+^2 on S^2. If $P \to L_1$, then for the corresponding point $\varphi^\circ(P)$

$$\varphi_1^\circ(P) \to +\infty, \quad \varphi_2^\circ(P) \to \text{const}, \quad \varphi_3^\circ(P) \to \text{const}$$

and then from $P \to L_3$ follows

$$\varphi_1^\circ(P) \to \text{const}, \quad \varphi_2^\circ(P) \to \text{const}, \quad \varphi_3^\circ(P) \to +\infty$$

and from $P \to L_2$ follows

$$\varphi_1^\circ(P) \to \text{const}, \quad \varphi_2^\circ(P) \to +\infty, \quad \varphi_3^\circ(P) \to \text{const}$$

If $P \to R_i$, then $\varphi_j^\circ \to +\infty$ $(i, j = 1, 2, 3)$. Let us consider the image of the cone consisting of rays passing through a piece of the surface S_+^2, bounded by the contour (1234561), and symmetric with respect to the straight line $\alpha = \beta = \gamma$. As $\nu \to 0$ the image of each ray abuts the point $\varphi^\circ(\alpha, \beta, \gamma)$ of surface (3.1) (with $\nu = 0$); when $\nu \to +\infty$, all the rays, as noted above, have the common tangent $\varphi_1 = \varphi_2 = \varphi_3$. The desired image of the contour is a geometric figure symmetric relative to line (4.1), resembling a three-petalled flower growing from the origin $\varphi_1 = \varphi_2 = \varphi_3 = 0$ in the direction of its axis (4.1). The flower's petals comprise a piece of surface (3.1) (with $\nu = 0$), bounded by the image of contour (1234561).

Examining the intersection of surface $\varphi^\circ(S_+^2)$ with the cone $\pi^{-1}(S)$ (it is shaded in Fig.4) and taking into account the correspondence of boundaries shown in Fig.4, we can obtain a central projection of the stability domain in space $(\nu, \alpha, \beta, \gamma)$ onto S_+^2. It is shown in Fig.5, where, as in Fig.4, S_+^2 is depicted as an equilateral triangle with vertices R_1, R_2, R_3. The segment R_3T corresponds to ellipsoids of revolution $(\alpha = \beta)$ and the point O (the center of S_+^2) corresponds to a spherical planet $(\alpha = \beta = \gamma)$. The shaded triangle R_2R_3T is the stability domain's projection mentioned. The cross-hatching denotes a domain (MNR_3) in which first approximation stability obtains for all $0 < \nu < +\infty$ and, hence, for all admissible ρ. The single shading corresponds to rays defined by the points on S_+^2 at which stability holds, but not everywhere for $\nu \in (0, +\infty)$, i.e., a reorganization of the types of the singular point P_1 takes place as ν grows along a ray. The curve MN separating these domains is shown approximately in Fig.5. It is important only that it be located at a finite distance from point O and from the interval (OR_3) corresponding to prolate ellipsoids of revolution.

We select sufficiently narrow cones C_φ and C' enclosing the lines (4.1) and $\alpha = \beta = \gamma$, such that $\varphi(C') \subset C_\varphi$. This can be done by virtue of a) the continuity of mapping φ and b) the fact that the images of all rays are tangent to line (4.1). As a matter of fact, from property b) it follows that for every cone $C \subset \mathbf{R}_+^3$ there exists $\nu_0 > 0$ such that the image of set $C \cap \{(\nu, \alpha, \beta, \gamma)\ \nu > \nu_0\}$ lies in C_φ when $\nu > \nu_0$. Further, from analysis we know the simple

Assertion. The convergence

$$f(x, y) \to f(x, y_0), \quad y_0 \in G, \quad y \to y_0$$

holds uniformly on K for a continuous mapping $f: K \times G \to \mathbf{R}^3$, where K is a compact space and G is an open set (K and G are assumed imbedded in some metric space).

We set $f = \varphi$, $K = [0, \nu_0]$, $G = \{(\alpha, \beta, \gamma): \alpha^2 + \beta^2 + \gamma^2 = 1,\ \alpha > 1/\sqrt{3} - \delta,\ \beta > 1/\sqrt{3} - \delta,$

$$\gamma > 1/\sqrt{3} - \delta\}$$

where δ is sufficiently small, $y_0 = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$; $K \subset \mathbf{R}_+^1 = \{x : x > 0\}$, $G = S_+^2$, where $\mathbf{R}_+^1$ and S_+^2 have the natural structure of metric spaces. Since $\varphi: \mathbf{R}_+^3 = \mathbf{R}_+^1 \times S_+^2 \to \mathbf{R}^3$, we can apply the Assertion made above. Consequently, we can choose a sufficiently narrow cone $C' \subset C$ enclosing the line $\alpha = \beta = \gamma$, such that the images of all its component rays lie in C_φ when $\nu \in [0, \nu_0]$. But for $\nu > \nu_0$ they stay in C_φ a fortiori. In $\mathbf{R}_+^3$ we introduce the structure of a direct product; then we can represent it as an octant from which we throw out the intersection with a ball centered at the origin. In this connection the part of the boundary, specified by the equality $\nu = 0$, is a spherical triangle S_+^2. From formula (3.1) we see that for corresponding points $\alpha \gtrless \beta$ equivalently $\varphi_1 \lessgtr \varphi_2$. Therefore, if $\alpha < \beta$

for a ray, then $\varphi_1 > \varphi_2$ for its whole image, i.e., stability holds for all $\nu > 0$. Analogously, when $\alpha > \beta$ instability holds for all $\nu > 0$. However, this conclusion is true only in C'. Thus, dimension arguments, as well as the geometric conclusions presented, enabled us to prove the following theorem on the stability property.

Theorem. For an ellipsoidal planet, sufficiently close in form to a sphere, the libration point's property of being stable in the first approximation depends only on the ellipsoid's form and does not depend on the mass, the linear sizes, and the planet's angular velocity of rotation. As a matter of fact, for fixed α, β, γ sufficiently close to $1/\sqrt{3}$ stability or instability is preserved under any admissible ρ and, hence, for arbitrary M, l, ω.

Analogous reasonings enable us to prove this statement for ellipsoid close in form to prolate ellipsoids of revolution $(\alpha = \beta < \gamma)$.

5. Discussion of the results. Let us consider the connection of the parameters $(\nu, \alpha, \beta, \gamma)$ introduced with the parameters used in /1—4/ for investigating the stability of libration points P_i (Fig.1). In the papers mentioned the characteristic size a_0 was introduced from the relation $fM/(a_0^3\omega^2) = 1$, which permitted the Lagrange function to be represented as

$$L = {}^1/_2\,(\xi^{\cdot 2} + \eta^{\cdot 2} + \zeta^{\cdot 2}) + (\xi\eta^{\cdot} - \eta\xi^{\cdot}) + {}^1/_2\,(\xi^2 + \eta^2) + V$$

in the dimensionless variables $\xi, \eta, \zeta : x = a_0\xi, y = a_0\eta, z = a_0\zeta$. In this case the force function is /6/

$$V = \frac{3}{4}\int\limits_{\mu'}^{+\infty}\left(1 - \frac{\xi^2}{\alpha' + u} - \frac{\eta^2}{\beta' + u} - \frac{\zeta^2}{\gamma' + u}\right)\frac{du}{[(\alpha' + u)(\beta' + u)(\gamma' + u)]^{1/2}}$$

$$\alpha' = (l/a_0)^2\alpha^2,\ \beta' = (l/a_0)^2\beta^2,\ \gamma' = (l/a_0)^2\gamma^2,\ \ \xi^2/(\alpha' + \mu') + \eta^2/(\beta' + \mu') + \zeta^2/(\gamma' + \mu') = 1$$

In /1/ it was assumed that the squares of the principal semiaxes can be represented as

$$a^2 = R^2 + 10a_0^2\,\alpha_0/3,\ b^2 = R^2 + 10a_0^2\,\beta_0/3,\ c^2 = R^2 + 10a_0^2\gamma_0/3$$

where R is the radius of a planet having the same volume as the ellipsod. The latter condition yields

$$[(R^2 + 10a_0^2\alpha_0/3)\ (R^2 + 10a_0^2\beta_0/3)\ (R^2 + 10a_0^2\,\gamma_0/3)]^{1/2} = R^3 \qquad (5.1)$$

The relations $\rho = 1/(2\varphi_1) = 3fM/(4\omega^2 l^3)$, $fM/(\omega^2 a_0^3) = 1$ enable us to determine l, $a_0 = (3\varphi_1/2)^{1/3}$, where $\varphi_1 = F(\nu, \alpha^2) = \varphi_1(\nu, \alpha, \beta, \gamma)$. Hence, setting $R^2/a^2_0 = \nu_0$, we obtain formulas for the mapping $(\nu_0, \alpha_0, \beta_0, \gamma_0) \to (\nu, \alpha, \beta, \gamma)$

$$\Phi\lambda^2 = \nu_0 + 10\lambda_0/3 = \lambda',\ \lambda = \alpha, \beta, \gamma, \quad \Phi = [3\varphi_1(\nu, \alpha, \beta, \gamma)/2]^{2/3}$$

The parameters $(\nu_0, \alpha_0, \beta_0, \gamma_0)$ used in /1/ (the parameter ν_0 is not explicitly defined), just as $(\nu, \alpha, \beta, \gamma)$, are dependent. Indeed, condition (5.1) implies the dependency

$$(1 + 10\alpha_0/3\nu_0)\ (1 + 10\beta_0/3\nu_0)\ (1 + 10\gamma_0/3\nu_0) = 1 \qquad (5.2)$$

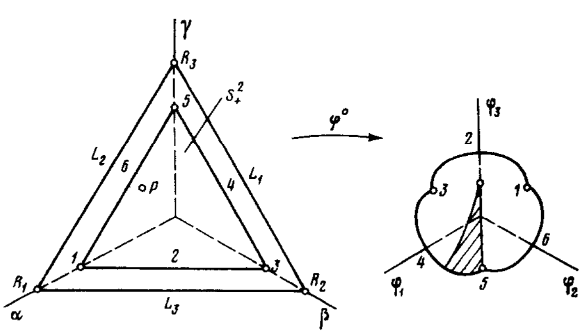

Fig.4

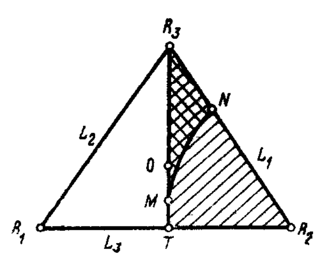

Fig.5

In /1/ the stability domain was obtained as follows. For a fixed ν_0 and with the use of power expansions with respect to $\alpha_0, \beta_0, \gamma_0$ it was proved that for sufficiently small $\alpha_0, \beta_0, \gamma_0$

$$|\alpha_0| < \varepsilon(\nu_0), \ |\beta_0| < \varepsilon(\nu_0), \ |\gamma_0| < \varepsilon(\nu_0) \qquad (5.3)$$

this domain is specified by the inequality $\alpha < \beta$. However, such a method does not yield a uniform estimate for the admissible limits of variation $\varepsilon(\nu_0)$ for all ν_0 and, consequently, does not permit the theorem proved above to be obtained. For this it is necessary to know the geometry of the whole domain of stability in-the-large and not just local parts of it. The dependency $\alpha_0 + \beta_0 + \gamma_0 = 0$ used in /1/ instead of (5.2), obtainable from (5.2) with due regard to terms linear in $\alpha_0, \beta_0, \gamma_0$, is admissible under (5.3) only for a sufficiently small ε depending on ν_0. At the same time the parameters $(\nu, \alpha, \beta, \gamma)$ enable us to achieve a result uniform in ν rapidly and intuitively.

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